Analytic Pricing of Employee Stock Options

By Jakša Cvitanić, Zvi Wiener, and Fernando Zapatero

Chongyu Li

Employee Stock Options (ESOs)

- The right to buy a certain amount of company shares at a predetermined price for a specific period of time
- Since the mid-1980s, stock options became a very popular choice for the compensation packages
- In 1999, 94% of companies in the S&P 500 offered stock options to their top employees

Standard Setting

1995

- FASB (with FAS 123)
- immediate recognition upon granted
- Intrinsic value
- Fair value (encourage)

2004

- FASB (with FAS 123R)
- Fair value only
- IFRS states the same principle

2005

- SEC (SAB 107)
- Provide guidance for fair value method

Fair Value Criterion of SAB 107

- Consistent with the fair value objective
- Based on established principles of financial economic theory
- Reflects all substantive characteristics of the instrument

Characteristics of the Stock Options

- Long-term (up to 10 years)
- Vesting periods of up to 4 years
- American type
- In the case of employee leaving the firm or being fired
  - Before vesting, the options are forfeited
  - After vesting, the employee has a short time to exercise the option
- Not transferrable and are restricted from hedging

Pricing Model

- rate of exit (intensity of Poisson Process) – λ
- stock price barriers - Level L and rate of decay α
- maturity – T
- strike price – K
- Black & Scholes framework
- We assume the stock price follows a lognormal process:
  $dS_t / S_t = \mu dt + \sigma dW_t$
  $S_0 = s$
- Under the risk neutral pricing measure, becomes
  $dS_t / S_t = r\, dt + \sigma dW_t$

Case A

- No vesting period, and the option is exercised when the stock price hits the desired level
- $L_t = L\, e^{\alpha t}$
- $L_t > K$ for $t \leq T$
- $\tau = T_L := \inf\{\, t > 0,\ S_t \geq L_t\} = \inf\{\, t > 0,\ S_t\, e^{-\alpha t} \geq L\}$
- The option price is equal to,
- $P_1 + P_2 := E\,[e^{-rT}\,(S_T - K)^+\, 1_{\{\tau > T\}}]$
  $+E\,[(L\, e^{(\alpha - r)\tau} - K\, e^{-r\tau})\, 1_{\{\tau \leq T\}}]$

Case B

- No vesting period, and the option is exercised when the employee leaves the company or is fired

$$\frac{f(t)}{1 - F(t)} = \lambda$$

$$\frac{F'(t)}{1 - F(t)} = \lambda$$

$$\frac{\left(1 - F(t)\right)'}{(1 - F(t)} = -\lambda$$

$$dlog\left(1 - F(t)\right) = -\lambda dt$$

$$\log\left(1 - F(t)\right) = a - \lambda t$$

$$1 - F(t) = a^* e^{-\lambda t}$$

$$F(t) = 1 - a^* e^{-\lambda t}$$

$$F(0) = 0 \Rightarrow a^* = 1$$

$$F(t) = 1 - e^{-\lambda t}$$

Case B

- No vesting period, and the option is exercised when the employee leaves the company or is fired
- Conditional distribution of the exercise time is

$F(t) = 1 - e^{-\lambda t}$ Probability of exit

$1 - F(t) = e^{-\lambda t}$ Probability of survival

- The option price is equal to

$$E[\int_0^T \lambda(s_t - K)^+ e^{-(r+\lambda)t} dt + (S_T - K)^+ e^{-(r+\lambda)T}$$

Case C

- No vesting period, and the option is exercised when the employee leaves the company or is fired, or the stock price hits the desired level
- the exercise time is τ= min($T_{L,}$ T_λ)

$$J_1 + J_2 + J_3 = E[(Le^{(\alpha-\lambda-r)T_L} - Ke^{-(r+\lambda)T_L})1_{\{T_L \le T\}}]$$
$$+E[\int_0^T \lambda e^{-(r+\lambda)t}(S_t - K)^+ 1_{\{T_L > t\}} dt]$$
$$+E[e^{-(r+\lambda)t}(S_t - K)^+ 1_{\{T_L > T\}}$$

Case D

- Combined model with a vesting period
- Vesting period $[0, T_0]$
- Within the vesting period, the intensity of quitting, being fired is λ_0
- After the vesting period, the intensity is λ
- The employee will exercise when the stock price reaches the desired level $Le^{\alpha(t-T_0)}$
- We denote T_λ^0 by the time of quitting/being fired, and $T_L^0 = \min\{t \in [T_0, T] | S_t \geq Le^{\alpha(t-T_0)}\}$

Case D

- As before, we find that

$$F(t) = 1 - e^{-\lambda_0 t} 1_{\{T_L^0 > t\}}, t \leq T_0$$

and

$$F(t) = 1 - e^{-\lambda_0 T_0 - \lambda(t - T_0)} 1_{\{T_L^0 > t\}}, t > T_0$$

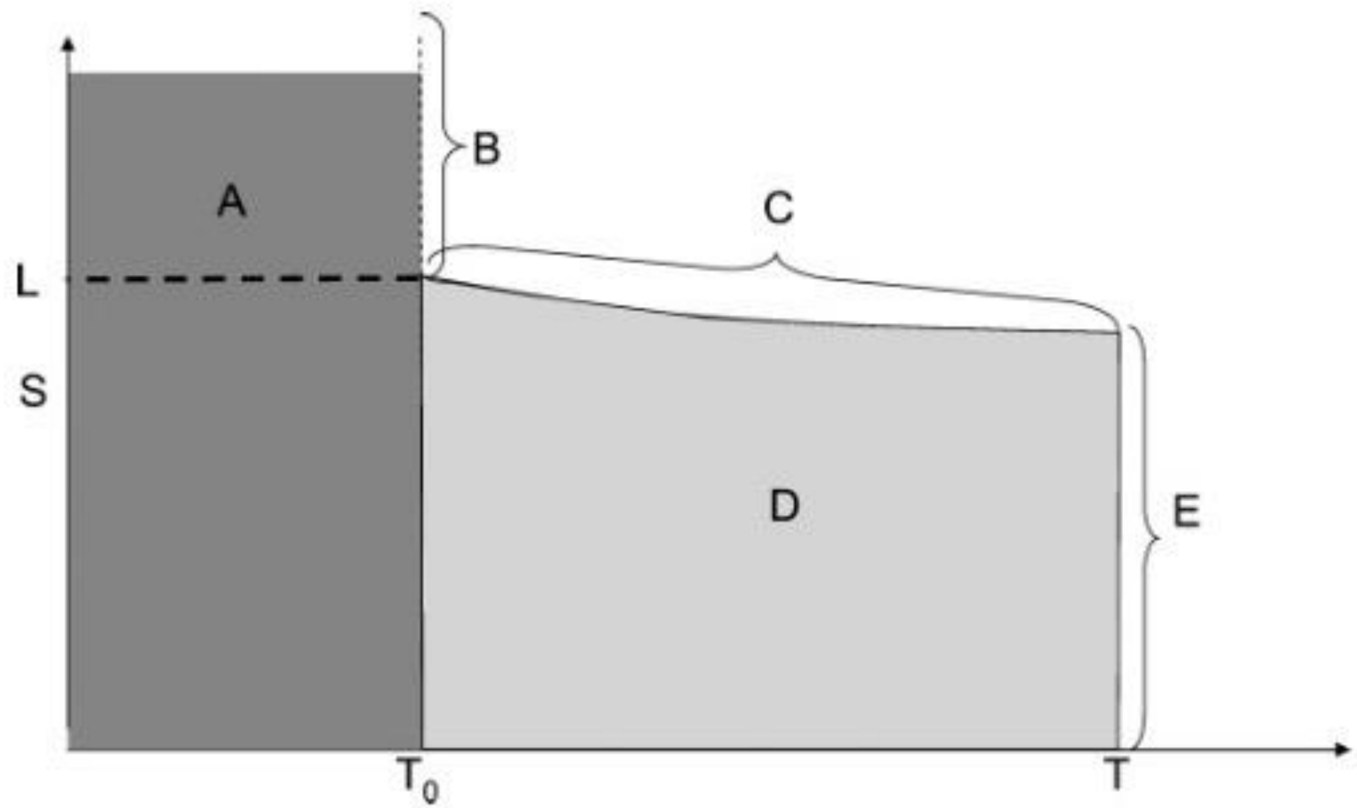
- Therefore, we get that the price is equal to

$$K_{11} + K_{12} + K_2 + K_3$$
$$= e^{(\lambda-\lambda_0)T_0} (E[(Le^{-\alpha T_0 - (r-\alpha-\lambda)T_L^0} - Ke^{-(r+\lambda)T_L^0}) 1_{\{T_L^0 \leq T, S_{T_0} < L_{T_0}\}}]$$
$$+ e^{(\lambda-\lambda_0)T_0} (E[(e^{-(r+\lambda)T_0} (S_{T_0} - K)^+ 1_{\{S_{T_0} \geq L_{T_0}\}}]$$
$$+ e^{(\lambda-\lambda_0)T_0} E[\int_{T_0}^{T} \lambda e^{-(r+\lambda)t} (S_t - K)^+ 1_{\{T_L^0 > t\}} dt]$$
$$+ e^{(\lambda-\lambda_0)T_0} E[e^{-(r+\lambda)T} (S_T - K)^+ 1_{\{T_L^0 > T\}}]$$

Limitations and assumptions

- The possibility of resetting
- Reloading: the provision that more options will be granted when the options of the initial package are exercised
- Dilution effect
- Possibility of default
- Continuous dividend payment
- Constant volatility
- Constant interest rate

Expiration of ESO – the probable scenarios



$$P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

Comparison with the binomial tree method

Table 1
Convergence of the binomial tree approach

"True" price is 27.8551

N	*P*
50	29.1894
100	29.0063
250	28.8949
500	28.4249
750	28.1550
1000	28.2934
1250	28.0380
1500	27.9424
1750	27.9404
2000	27.9973
2250	28.0925
2500	28.2135
3000	28.1587
4000	27.9921
5000	28.0327
7500	27.9592
10000	28.0239
20000	27.9003
40000	27.9291

Parameter values are:

$s = 100$

$K = 100$

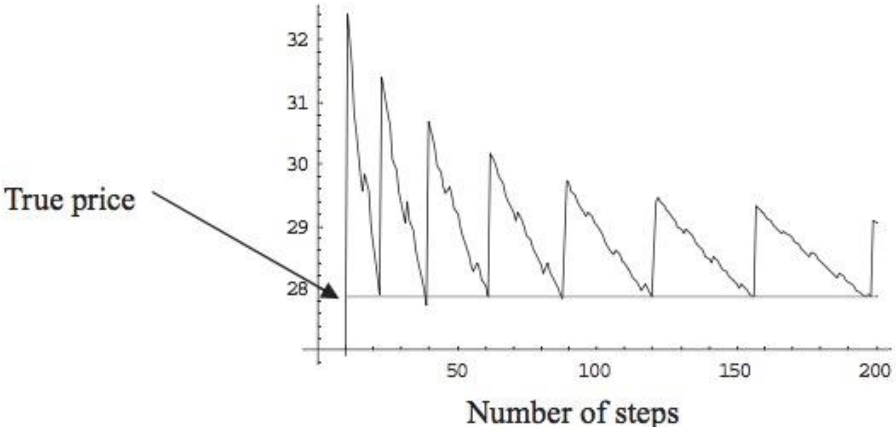
$T = 10$

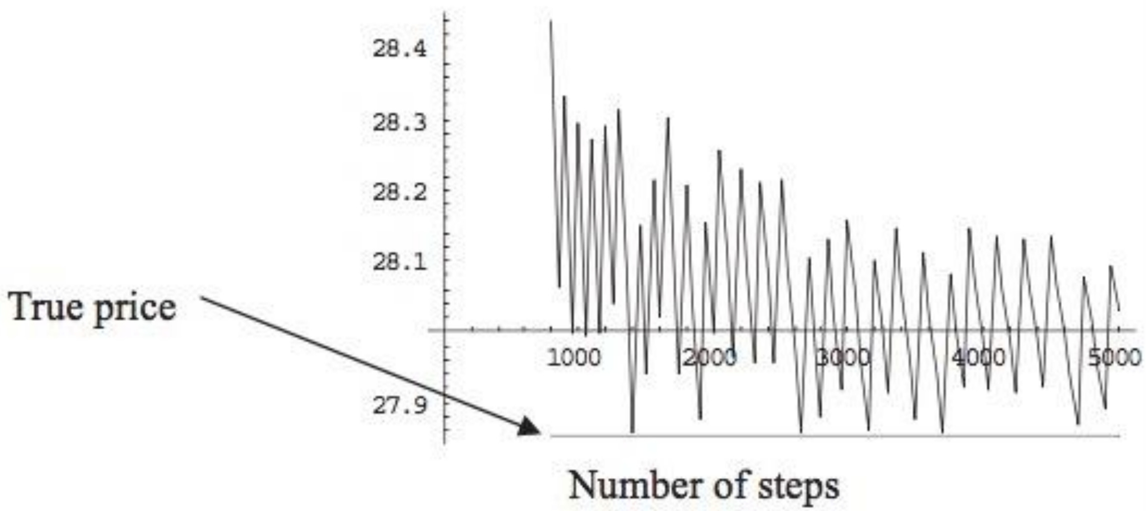
$T_0 = 2$

$\sigma = 0.2$

$R = 0.06$

Comparison with the binomial tree method





Price of ESOs for different parameter values

Prices of ESOs for different parameter values

	$T_0 = 3$; $\lambda = 0.04$					
	s	A	B	C	D	BS
$L = 125$	100	16.1088	38.9753	15.3372	22.7792	45.1930
	120	23.1375	56.4807	22.9921	35.7948	63.0836
$L = 150$	100	26.0510	38.9753	23.9052	26.8375	45.1930
	120	35.3827	56.4807	34.0925	39.2417	63.0836

	$T_0 = 3$; $\lambda = 0.2$					
	s	A	B	C	D	BS
$L = 125$	100	16.1088	24.4350	12.9962	13.5253	45.1930
	120	23.1375	41.1104	22.5402	21.7856	63.0836
$L = 150$	100	26.0510	24.4350	18.1005	15.2048	45.1930
	120	35.3827	41.1104	30.5150	23.2637	63.0836

	$s = 120$; $L = 125$					
	T_0	A	B	C	D	BS
$\lambda = 0.04$	1	23.1375	56.4807	22.9921	29.2254	63.0836
	3	23.1375	56.4807	22.9921	35.7948	63.0836
$\lambda = 0.2$	1	23.1375	41.1104	22.5402	24.2800	63.0836
	3	23.1375	41.1104	22.5402	21.7856	63.0836

	$s = 100$; $L = 150$					
	T_0	A	B	C	D	BS
$\lambda = 0.04$	1	26.0510	38.9753	23.9052	24.5668	45.1930
	3	26.0510	38.9753	23.9052	26.8375	45.1930
$\lambda = 0.2$	1	26.0510	24.4350	18.1005	17.4525	45.1930
	3	26.0510	24.4350	18.1005	15.2048	45.1930

Case Study

Characteristics of option grants of TEVA

# of grant	Date	# of options	$s = K$	T	$\% T_{2y}$	$\% T_{3y}$	$\% T_{4y}$	δ
1	7/23/2001	845,000	65.33	5	1/4	1/4	1/2	0.1%
2	2/14/2002	800,000	60.41	7	1/4	1/4	1/2	0.14%
3	3/24/2003	2,000,000	40.40	7	1/3	1/3	1/3	0.35%
4	7/12/2004	2,019,000	33.27	7	1/3	1/3	1/3	0.53%

Grant prices

	#	σ(%)	BS	SBS	BM	AF
			$\lambda = 0.1$			
$L = 2.5$	1	42.18	23.9921	15.6037	16.5567	16.6647
	2	41.00	24.4120	14.8436	15.8963	15.9173
	3	39.00	36.2617	22.1947	24.4876	24.7658
	4	37.20	29.5931	18.2149	19.9808	20.3841
$L = 2.0$	1	42.18	23.9921	15.6037	16.2487	16.3575
	2	41.00	24.4120	14.8436	15.3715	15.3461
	3	39.00	36.2617	22.1947	23.7564	23.9036
	4	37.20	29.5931	18.2149	19.2769	19.6506
			$\lambda = 0.15$			
$L = 2.5$	1	42.18	23.9921	13.2898	13.9910	14.0878
	2	41.00	24.4120	12.6476	13.2744	13.3024
	3	39.00	36.2617	19.1473	20.6329	20.8796
	4	37.20	29.5931	15.7139	16.8303	17.1775
$L = 2.0$	1	42.18	23.9921	13.2898	13.7382	13.8354
	2	41.00	24.4120	12.6476	12.8664	12.8570
	3	39.00	36.2617	19.1473	20.0645	20.2074
	4	37.20	29.5931	15.7139	16.2833	16.6049

Conclusion and Some Opinions

- Analytic expression for pricing ESOs
- Dividend / forfeitures
- Advantages/limitations
- Comply with the SEC criterion
- Applicable in the real case
- Typo
- Logic is not very clear
- “True” price